OVERTURNING A TREE

1 What is the percentage of all women who underwent a mammography and

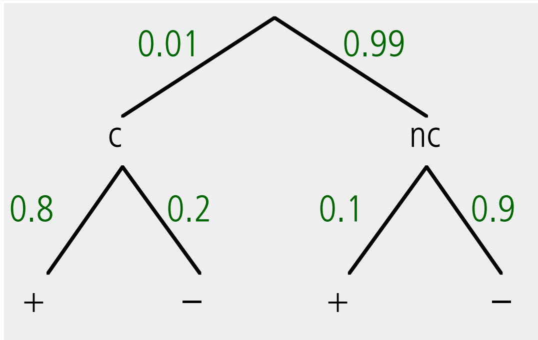
a) have cancer and are tested positive?
b) have cancer and are tested negative?
c) are tested positive and have no cancer?
d) are tested negative and have no cancer?

a) $p(c \cap +) = p(c) \cdot p(+ \mid c)$
$= 0.01 \cdot 0.8 = \mathbf{0.008} = \mathbf{0.8\ \%}$

b) $p(c \cap -) = p(c) \cdot p(- \mid c)$
$= 0.01 \cdot 0.2 = \mathbf{0.002} = \mathbf{0.2\ \%}$

c) $p(nc \cap +) = p(nc) \cdot p(+ \mid nc)$
$= 0.99 \cdot 0.1 = \mathbf{0.099} = \mathbf{9.9\ \%}$

d) $p(nc \cap -) = p(nc) \cdot p(- \mid nc)$
$= 0.99 \cdot 0.9 = \mathbf{0.891} = \mathbf{89.1\ \%}$



2 What is the probability of a woman who underwent a mammography and was tested negative to have breast cancer?

$$p(c \mid -) = \frac{p(c) \cdot p(- \mid c)}{p(-)} = \frac{0.01 \cdot 0.2}{0.01 \cdot 0.2 + 0.99 \cdot 0.9} \approx \mathbf{0.00224} = \mathbf{0.224\ \%}$$

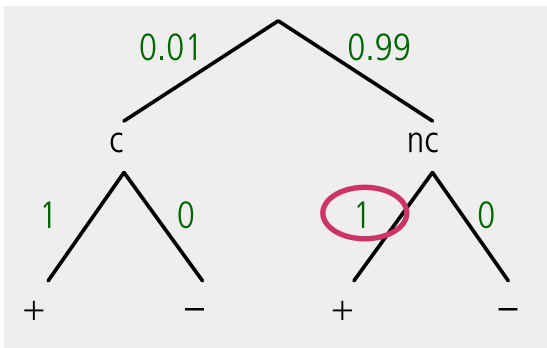
3 What is the probability of a woman who underwent a mammography and was tested negative to have breast cancer, calculated with tables?

	c	nc	total
+	8	99	107
−	2	891	893
total	10	990	1000

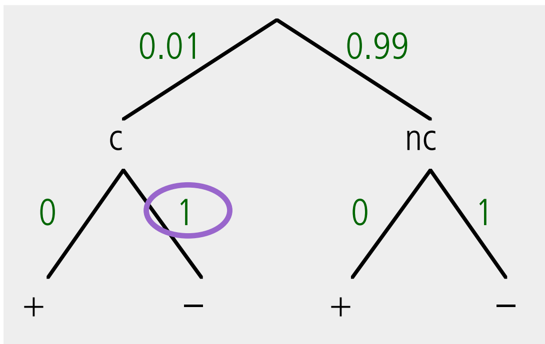
$$p(c|-)=\frac{2}{893}\approx \mathbf{0.00224}=\mathbf{0.224\ \%}$$

4 Prove the statement above!
Breast cancer attacks 1 % of all women between 40 and 60.
a) A fictional test declares all patients positive. Calculate its false positive rate.
b) A fictional test declares all patients negative. Calculate its false negative rate.

a) $p(+|nc)=\mathbf{1}$



b) $p(-|c)=\mathbf{1}$

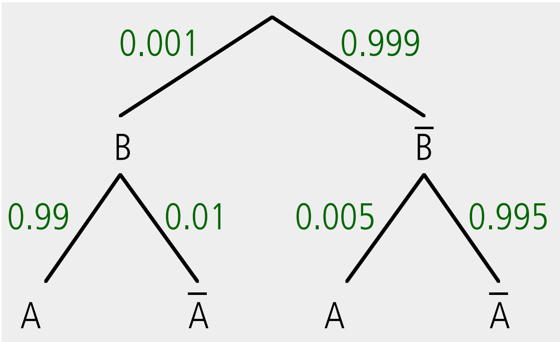


5 Statistics shows that a jewellery shop in a certain district of a town has a chance of 0.1 % to be robbed every night.
The owner decides to have an alarm system installed. If a burglary takes place the system alerts the police in 99 out of 100 cases. Unfortunately the system is so sensitive that it goes off without burglary too, but only once every 200 nights.
Evaluate the probability for one night that the system alerts the police.

The burglary and the alarm form a compound experiment with two levels.

The following events are needed:
B ="burglary is taking place"
A = "alarm goes off"

Each night a compound experiment is started:



$$p(A) = p(B)\cdot p(A\,|\,B) + p(\overline{B})\cdot p(A\,|\,\overline{B}) = 0.001\cdot 0.99 + 0.999\cdot 0.005 \approx \mathbf{0.006} = \mathbf{0.6\ \%}$$

6 Much more interesting for the police is the question:
The alarm goes off. What is the probability that a burglary is taking place?

$$p(B\,|\,A) = \frac{p(B)p(A\,|\,B)}{p(A)} = \frac{0.001\cdot 0.99}{0.001\cdot 0.99 + 0.999\cdot 0.005} \approx \mathbf{0.165} = \mathbf{16.5\ \%}$$

With rare events most alarms are false alarms!

7 A factory produces semiconductors. Experience shows that 1 out of 500 produced semiconductors does not work properly. To maintain the company's good reputation only "good" semiconductors should leave the factory. Therefore each semiconductor has to pass a final check. The "bad" ones are thrown into a bin, the "good" ones are packed to be sent to the customers.

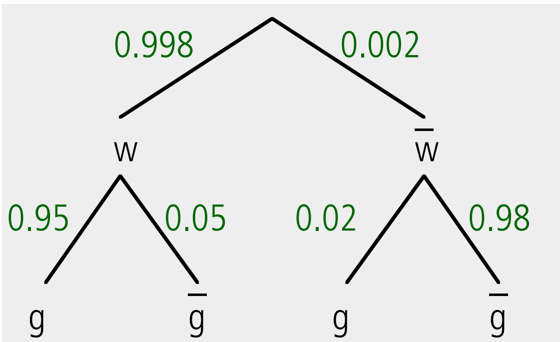
Nevertheless 2 % of the "bad" products pass the check and 5 % of the "good" ones end up in the bin.

a) What is the probability that a semiconductor that has not passed the check is "good"?
 In other words: What is the rate of the "good" ones in the bin?

b) What is the probability that a semiconductor that has passed the check is "bad"?
 In other words: What is the rate of the "bad" ones sent to customers?

Production and final check of each semiconductor form a compound experiment with two levels.

The following events are needed:
w ="semiconductor is working"
g = "passed the check as good"



a) $$p\left(w\,|\,\overline{g}\right)=\frac{p\left(w\right)p\left(\overline{g}\,|\,w\right)}{p\left(\overline{g}\right)}=\frac{0.998\cdot 0.05}{0.998\cdot 0.05+0.002\cdot 0.98}\approx \mathbf{0.962}=\mathbf{96.2\ \%}$$

Nearly all the semiconductors in the bin are actually working!

b) $$p\left(\overline{w}\,|\,g\right)=\frac{p\left(\overline{w}\right)p\left(g\,|\,\overline{w}\right)}{p\left(g\right)}=\frac{0.002\cdot 0.02}{0.002\cdot 0.02+0.998\cdot 0.95}\approx \mathbf{0.000042}=\mathbf{0.0042\ \%}$$

The clients of this factory will be happy. Practically all semiconductors delivered are working.

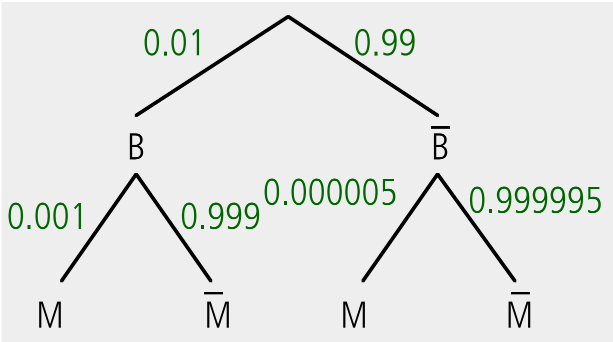
8

To check on Good's result a model has to be created. Assume the following:

1) 100 people, her husband included, had access to the victim, and all are equally likely to have been present at the exact time of her death.
2) The probability that a beating husband kills his wife is 0.1 % (as indicated by Dershowitz), that of the average person to kill a wife is 0.000005.

Calculate the probability that O. J. Simpson has killed his wife under the condition that his wife is dead.

The choice of the person present at the moment in question and whether this person kills or not form a compound experiment with two levels.



The following events are needed:
B ="the beating husband is present"
M = "murder is committed"

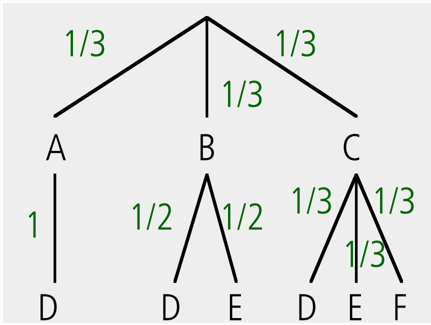
$$p(B|M)=\frac{p(B)p(M|B)}{p(M)}=\frac{0.01\cdot 0.001}{0.01\cdot 0.001+0.99\cdot 0.000005}\approx \mathbf{0.669}=\mathbf{66.9\ \%}$$

With this model it is even more likely that O. J. Simpson killed his wife than Good calculated.

9

There are three identical boxes with cookies stored in the basement. Box A is filled with brownies only, box B with equal amounts of brownies and butter fingers, all mixed, box C with equal amounts of brownies, butter fingers and ginger breads, all mixed again.
One evening in the dark you sneak down to the basement, you choose a box and you grab the first cookie you feel, all completely at random. Up in your room you realise that you have picked a brownie. What is the probability that you opened box A?

The choice of the box and the choice of the cookie form a compound experiment with two levels.



The following events are needed:
A ="box A is picked"
B = "box B is picked"
C = "box C is picked"
D = "brownie is grabbed"
E = "butter finger is grabbed"
F = "ginger bread is grabbed"

$$p(A|D)=\frac{p(A)p(D|A)}{p(D)}=\frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1+\frac{1}{3}\cdot\frac{1}{2}+\frac{1}{3}\cdot\frac{1}{3}}=\mathbf{\frac{6}{11}}\approx \mathbf{54.5\ \%}$$